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Surface Interpolation of Non-four-sided and Concave Area by NURBS Boundary Gregory Patches

Junji Sone, Kouichi Konno, and Hiroaki Chiyokura

Abstract. Subdivision methods are widely used for surface interpolation of a non-four-sided area. Using this method, smooth surface shape control of multiple surfaces is difficult. Therefore, we used the trim surface for interpolation of a concave area, but shape control of the trim surface was also difficult. In this research, the surface interpolation method of non-four-sided (over 4 sided) area uses a single NBG (NURBS Boundary Gregory) patch. Moreover we apply this method for concave areas and study the ability of interpolation. Interpolation of one-sided and two-sided concave shapes is also considered. As a result, one-sided concave area can be interpolated smoothly for complex cases. Two-sided cases need further study.

§1. Introduction

Catmull-Clark [1] and Doo-Sabin [5] subdivisions are widely used to interpolate a non-four-sided area. This method increases the number of surfaces. Designers must modify the non-four-sided area, observing the contour curves, silhouette pattern and highlight lines [13] when modeling the outer shape of products. Surface shape control is more difficult for multiple surfaces, which are generated by subdivision, because it corrects several surfaces at the same time. Moreover, if we apply Catmull-Clark subdivision to concave areas, part of the inner surface twists and protrudes from the boundary curves. Then, the trimmed surface is commonly used for concave area interpolation. Boundary curve of trimmed surface must coincide with inner surface shape. If we modify the trimmed boundary curves, surface shape must follow the change in boundary curves precisely. This modification is very difficult. The Vertex Blending method [6,2] is proposed to interpolate the non-four-sided area by a single patch. It is also difficult to apply this method for concave area interpolation.

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In this paper, we propose the surface interpolation method using a single NURBS boundary Gregory(NBG) [14,8] patch for non-four-sided areas and concave areas. Applying this method to concave area interpolation, selection of the four edges is important. Our basic idea is that smooth streamlines make smooth surfaces. Smoothness of streamlines would be influenced by the construction of four edges. Therefore, we propose the evaluation parameters to evaluate the smoothness of streamlines. Then, we develop the edge construction method using evaluation parameters and apply this method to one and two-sided of the concave area. As a result, single NBG patch can interpolate non four-sided-area, including concave shapes, allowing the surface shape to be modified flexibly.

§2. Surface Construction of NBG Patch for Non-four-sided Areas

2.1 Abstract of NURBS boundary Gregory patch

Boundary curves of the NBG patch are expressed as a NURBS [15] curve. This patch is an extended general boundary Gregory patch [9], constructed from three sub-patches and calculated by following equation:

$$\mathbf{S}(u,v) = \mathbf{S}^u(u,v) + \mathbf{S}^v(u,v) - \mathbf{S}^c(u,v). \tag{1}$$

$\mathbf{S}^u$ is defined by the boundary curves $\mathbf{S}(0,v)$ and $\mathbf{S}(1,v)$ and their cross boundary derivatives $\mathbf{S}_u(0,v)$ and $\mathbf{S}_u(1,v)$. The boundary curves and the cross boundary derivatives are expressed by a NURBS. Similarly, $\mathbf{S}^v$ is represented by boundary curves $\mathbf{S}(u,0)$ and $\mathbf{S}(u,1)$ and the cross boundary derivatives $\mathbf{S}_v(u,0)$ and $\mathbf{S}_v(u,1)$. $\mathbf{S}^c$ is called a common surface $\mathbf{S}^u$ and $\mathbf{S}^v$, which is a cubic rational boundary Gregory patch [3]. Detailed surface construction is described in the reference paper [14,8].

2.2 Abstract of surface interpolation of non-four-sided areas

In this subsection, a surface interpolation method using NBG patch for non-four-sided (over 4 sided) areas is described. We explain the continuity correction method in the next section.

2.2.1 Sub-patch generation of $\mathbf{S}^u$

Fig. 1 shows an interpolation method of pentagonal area. Here the u-direction order of $\mathbf{S}^u$ and the v-direction order of $\mathbf{S}^v$ are cubic. The subdivision method of $\mathbf{S}^u$ is described in this figure. First, the new point $\mathbf{P}_{12}$ is generated by dividing the edge at the same parameter point of C^0 vertex $\mathbf{P}_{11}$. Next, a straight line is generated from $\mathbf{P}_{11}$ to $\mathbf{P}_{12}$. The vector $\mathbf{V}^{c1}$ is calculated by multiplying 1/3 to the vector from $\mathbf{P}_{11}$ to $\mathbf{P}_{12}$. The plane $\mathbf{PL}^1$ is generated from the tangent vector of boundary curves $\mathbf{V}^{u11}$ and $\mathbf{V}^{u21}$. $\mathbf{V}^{u18'}$ is derived by projecting $\mathbf{V}^{c1}$ to plane $\mathbf{PL}^1$. If continuity is more than G^1 at point $\mathbf{P}_{12}$, $\mathbf{V}^{u17'}$ is calculated by a weighted average of the tangent vector $\mathbf{V}^{u14}$ and $\mathbf{V}^{u24}$ using the $\mathbf{P}_{12}$ parameter value. Occasionally, the length of $\mathbf{V}^{u17'}$ and $\mathbf{V}^{u18'}$

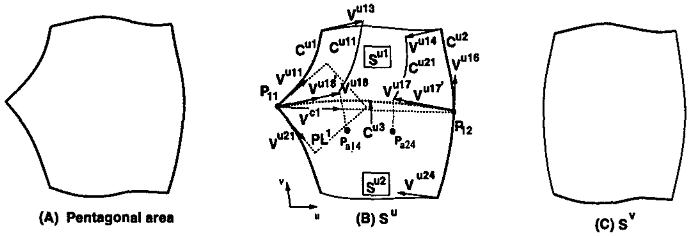

Fig. 1. Surface interpolation method for pentagonal area using a NBG patch.

are much different. In this case, the control polygon decided by $\mathbf{P}_{11}, \mathbf{P}_{12},,$ $\mathbf{V}^{u18'}, \mathbf{V}^{u17'}$ is compensated as equal lengths and new points $\mathbf{V}^{u18}, \mathbf{V}^{u17}$ are generated. The cubic Bézier curve $\mathbf{C}^{u3}$ is generated from the derived points and $\mathbf{S}^u$ is subdivided.

The sub-patch generation procedure is as follows. Firstly, the boundary curves $\mathbf{C}^{u1}$ and $\mathbf{C}^{u2}$ are converted to NURBS curves. The order of $\mathbf{C}^{u1}$ and $\mathbf{C}^{u2}$ are adjusted to the same value. Secondly, a quadratic Bézier form derivative function is generated from the vectors $\mathbf{V}^{u13}$, $\mathbf{V}^{u18}$ and the average of these vectors. This function is converted to NURBS, the degree is elevated to the same order, and knots are inserted as the same knot vectors of boundary curve $\mathbf{C}^{u1}$. Thirdly, $\mathbf{C}^{u11}$ can be derived by adding this derivative function to the boundary curve $\mathbf{C}^{u1}$. Similarly, $\mathbf{C}^{u21}$ can be computed from the vectors $\mathbf{V}^{u14}$,$\mathbf{V}^{u17}$ and the average of these vectors. Finally, the NURBS surface whose u-direction order is 4, is generated from four NURBS curves $\mathbf{C}^{u1}$,$\mathbf{C}^{u11}$, $\mathbf{C}^{u21}$ and $\mathbf{C}^{u2}$.

2.2.2 Continuity correction between sub-patches

Continuity of sub-patch $\mathbf{S}^{u1}$ and $\mathbf{S}^{u2}$ must be contained in G^1 for constructing G^1 continuity NBG patch. The quadratic derivative function is calculated from the tangent vectors $\mathbf{V}^{u11}$,$\mathbf{V}^{u16}$ of the boundary curve $\mathbf{S}^{u1}$ and the average of the tangent vectors $\mathbf{C}^{u11}$ and $\mathbf{C}^{u21}$. The inner control points of the sub-patch $\mathbf{S}^{u2}$ are modified using the G^1 connection method [4]. Through this procedure, the continuity of the sub-patches $\mathbf{S}^{u1}$ and $\mathbf{S}^{u2}$ can be made G^1. Finally a single NURBS surface $\mathbf{S}^u$ is generated by concatenating these sub-patches.

2.3 G^1 connection with surrounding surface

The non-four-sided surface is generated at the intersection area of three or more fillets. This non-four-sided surface must be joined to the surrounding surface with G^1 continuity. Sarraga [11] shows the G^1 continuity and twist compatibility condition around the vertex where N surfaces are joined. This research is limited to the pair of boundary curves that must be joined with G^1 continuity. However, in our case, the pair of boundary curves is not joined G^1. Therefore, we propose the blending method of twist vectors to satisfy

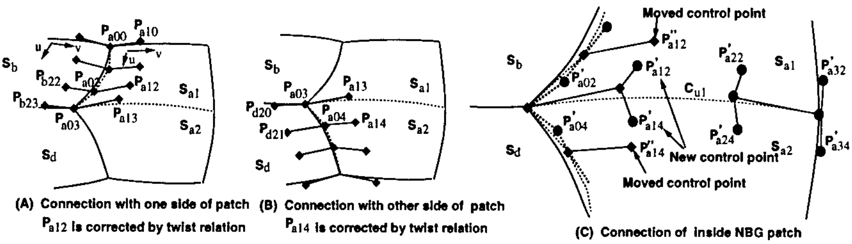

Fig. 2. G^1 connection with surround surface.

the twist compatibility conditions. The procedure to make the G^1 connection with surrounding surface is described below.

2.3.1 Cubic boundary curve

Fig. 2 shows the continuity correction method around a C^0 vertex. We explain that boundary curves are cubic NURBS and their weights are 1.0 which is the same as the cubic Bézier curve. The surface interpolation method for non-four-sided area $\mathbf{S}_a$ is described as follows: Firstly, the continuity between $\mathbf{S}_a$ and $\mathbf{S}_b$ is corrected to G^1 using quadratic Bézier cross boundary derivatives. The basic G^1 connection equation is expressed as

$$\frac{\partial \mathbf{S}_{a1}(u,0)}{\partial v} = k(u)\frac{\partial \mathbf{S}_b(u,1)}{\partial v} + h(u)\frac{\partial \mathbf{S}_{a1}(u,0)}{\partial u}. \tag{2}$$

Here, $k(u) = k_0(1-u) + k_1 u, h(u) = h_0(1-u) + h_1 u$.

The twist equation is generated by differentiating equation (2) with respect to u as follows:

$$\begin{aligned}\frac{\partial^2 \mathbf{S}_{a1}(u,0)}{\partial v \partial u} = k_u(u)\frac{\partial \mathbf{S}_b(u,1)}{\partial v} + k(u)\frac{\partial^2 \mathbf{S}_b(u,1)}{\partial v \partial u} +\\ h_u(u)\frac{\partial \mathbf{S}_{a1}(u,0)}{\partial u} + h(u)\frac{\partial^2 \mathbf{S}_{a1}(u,0)}{\partial^2 u}.\end{aligned} \tag{3}$$

The control point $\mathbf{P}_{a12}$ can be regenerated using twist compatibility equation (3). $\mathbf{P}_{a12}$ is calculated by applying the quadratic derivative function to equation (3) as follows:

$$\begin{aligned}\mathbf{V}_{a2} = \frac{1}{3}\Big[3\mathbf{V}_{a3} - \frac{n_{vb}(k_1-k_0)\mathbf{V}_{b2}}{3} + n_{vb}k_1(\mathbf{V}_{b2}-\mathbf{V}_{b1})\\ + (h_1-h_0)\mathbf{V}_{c2} + 2h_1(\mathbf{V}_{c2}-\mathbf{V}_{c1})\Big],\\ \mathbf{P}_{a12} = \mathbf{V}_{a2} + \mathbf{P}_{a03}.\end{aligned} \tag{4}$$

Here, n_{vb} is v-direction order of $\mathbf{S}_b$,

$$\mathbf{V}_{ai} = \mathbf{P}_{a1i} - \mathbf{P}_{a0i}, \mathbf{V}_{bi} = \mathbf{P}_{a0i} - \mathbf{P}_{b2i}, \mathbf{V}_{ci} = \mathbf{P}_{a0(i+1)} - \mathbf{P}_{a0i}. \tag{5}$$

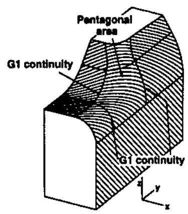

Fig. 3 G^1 connection with surrounding surface.

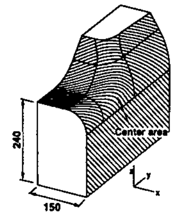

Fig. 4 Shape control result of center area.

By the same procedure, continuity between $\mathbf{S}_a$ and $\mathbf{S}_d$ is corrected to G^1 using quadratic Bézier cross boundary derivatives. $\mathbf{P}_{a14}$ is calculated by the same method to satisfy the twist compatibility. Next, continuity between sub-patches $\mathbf{S}_{a1}$ and $\mathbf{S}_{a2}$ are connected to G^1. Generally, it is difficult to satisfy twist compatibility conditions around the C^0 continuous vertex. Therefore, in order to solve this problem, new control points are generated by knot insertion and these control points are corrected to satisfy the twist compatibility. In this method, boundary curves and cross boundary derivative curves are converted NURBS. $\mathbf{P}'_{a02}$ is generated by knot insertion between $\mathbf{P}_{a02}$ and $\mathbf{P}_{a03}$. Knot vectors are computed by the following procedure:

Case 1: (the number of control points and the order are the same). If a knot of 0.5 is inserted, all the control points of the section will be regenerated by uniform distance. Then, we used 0.5 for the knot vector for both cases; generating a new control points at the start and end of the boundary curve.

Case 2: (piecewise boundary curve). By the same considerations, the knot is set to $knotv[order]/2.0$ for generating new control points at the start of the boundary curve. At the end of the boundary curve, the knot value is set to $1.0 - (1.0 - knotv[cnum - 1])/2.0$. Here, $knotv[]$ is knot vectors. $cnum$ is the number of control points.

These knot values should be adjusted by designer requirements. By the same procedure, $\mathbf{P}'_{a12}$,$\mathbf{P}'_{a22}$,$\mathbf{P}'_{a32}$, $\mathbf{P}'_{a04}$,$\mathbf{P}'_{a14}$,$\mathbf{P}'_{a24}$ and $\mathbf{P}'_{a34}$ can be derived. The control points $\mathbf{P}'_{a14}$ and $\mathbf{P}'_{a24}$ are calculated by the equation (4) to satisfy twist compatibility.

This method can be used to interpolate any number of non-four-sided area (over 4 sided). The G^1 continuity can be satisfied with the surrounding surface, and the twist compatibility condition can also be satisfied.

The surface shape of the boundary area and the center area can be modified smoothly by using the NBG patch shape control method. Fig. 3 shows a surface interpolation result for a pentagonal area. Continuity with neighboring surface is corrected to G^1. Contour curves of the surface are smooth. The degree of $\mathbf{S}^c$ is elevated to quartic and the center control point is moved 30 mm for surface normal direction at $u = 0.5, v = 0.5$. Fig. 4 shows the interpolation result. The center area of the surface can be modified to obtain G^1 continuity with the neighboring surfaces.

2.3.2 Case of piecewise and rational boundary curves

The Konno method [7] is used to join G^1 between piecewise $\mathbf{S}_{a1}$ and piecewise $\mathbf{S}_b$. If the boundary curve is of rational form, the Chiyokura method [3] is used. If the boundary curve is of rational form and the weights of $\mathbf{P}_{a03}$ and $\mathbf{P}'_{a02}$ are different, we should use the LIU method[10] to make a G^1 connection between $\mathbf{S}_{a1}$ and $\mathbf{S}_{a2}$.

§3. Concave Areas

In this section, the surface interpolation result is described for concave areas. Firstly, we apply it to a one-sided concave area. Next, we consider a two-sided concave area.

3.1 Basic idea

It is important to select the four side edges to interpolate the concave area. Our basic idea is that smooth surface have a smooth streamlines. We selected 3 parameters to evaluate the smoothness of streamlines as follows:

1) Max difference of isoparametric line length : MDI
2) Max difference of variation of isoparametric line length : MDVI
3) Max difference of isoparametric line width : MDIW

Here, 1) and 2) evaluate the variation of streamline length and, 3) evaluates the variation of the width of streamline. We believe that the surface can be smooth if these parameters take a small value. Designing four-side edges proceeds as follows:

1) Check concave area,
2) Decide the axis to make a symmetrical shape,
3) Surface edges are selected by symmetry, and concave area curves are composed as one edge,
4) Interpolate by NURBS boundary Gregory patch,
5) Evaluate the surface shape by the evaluation parameter,
6) Make the best edge selection.

3.2 Interpolation for a one-sided concave area

The surface interpolation method is applied to a one-sided concave area. Figs. 5–7 show the results of interpolation for a simple case. The concave area is generated by difference boolean operation to cubic, which is cut by the free form surface. Fig. 8 shows the evaluation results of the surface. Pattern 1 takes small parameter values and isoparametric lines and shading images are smooth as shown in these figures.

Figure 9 shows 3 types of surface interpolation results for deep concave areas which are modeled by the difference boolean operation for a cubic which is cut by free form surfaces. Figure 10 shows the evaluation results using

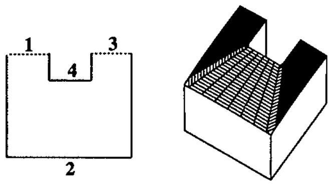

Fig. 5. One-sided concave area: pattern 1

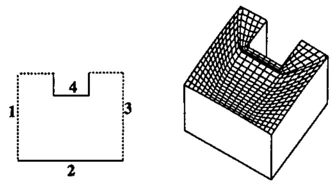

Fig. 6. Pattern 2.

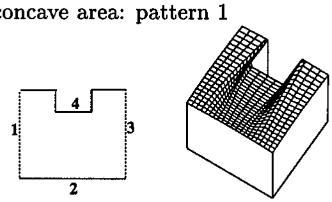

Fig. 7. Surface interpolation of one-sided concave area: pattern 3.

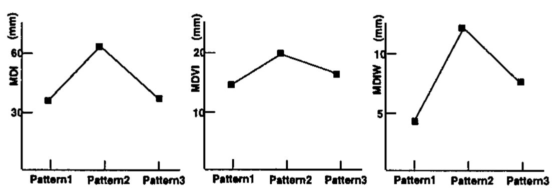

Fig. 8. Surface evaluation result.

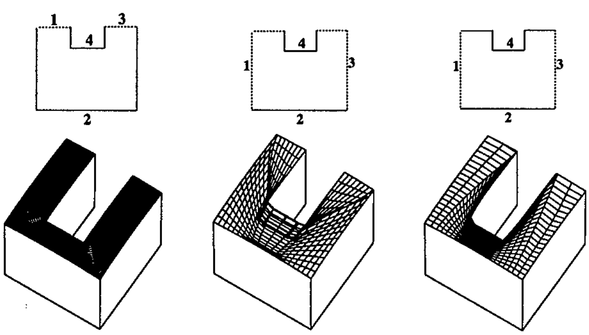

Fig. 9. Surface interpolation of one side concave area: large case.

the previous parameters. Pattern 1 takes small parameter values, and the isoparametric lines are smooth as shown in these figures.

Figure 11 a) shows surface isoparametric lines of complex concave area, b) is the shaded image, and c) shows the control points of S^u. The interpolated surface is smooth as shown in this figure. From these results, smooth surfaces can be generated by selecting the four-side edges to minimize the MDI, MDVI and MDIW. We can get smooth surface for complex concave area.

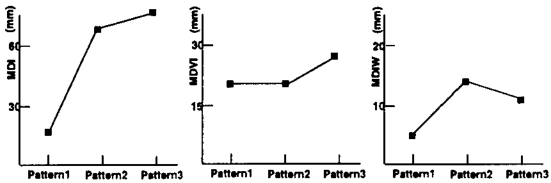

Fig. 10. Surface evaluation result.

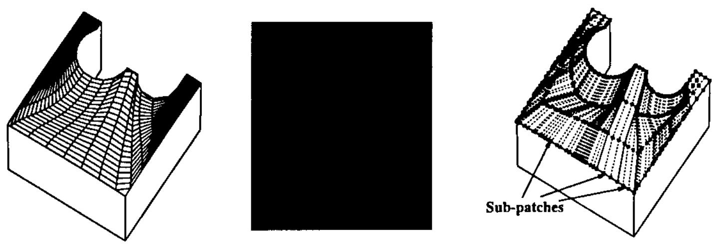

Fig. 11. a) Complex case b) Shading image c) Control points of S^u.

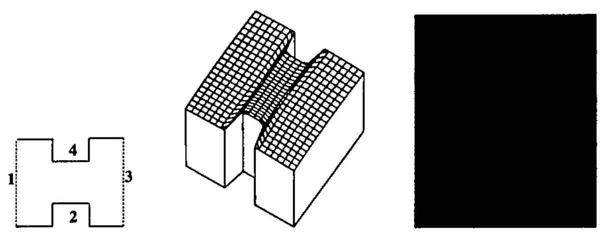

Fig. 12. Surface interpolation of two side concave area : pattern 1.

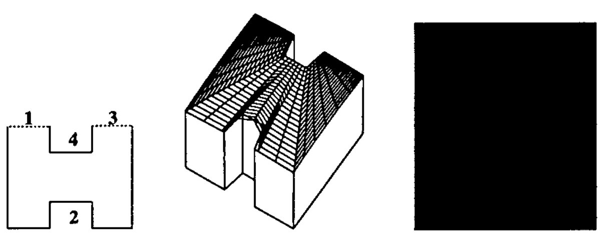

Fig. 13. Surface interpolation of two side concave area : pattern 2.

3.3 Two side of concave area

In the same manner, a simple two-sided concave area is modeled by the difference boolean operation for a cubic which is cut by a free form surface. Figs. 12-14 show the surface interpolation results. Fig. 15 shows the evaluation result for the generated surface. Although pattern 1 shows the lowest strain of the surface from the shaded image, it still has a strain in the middle area. In this case, MDI and MDIW are smaller for the low strain surface.

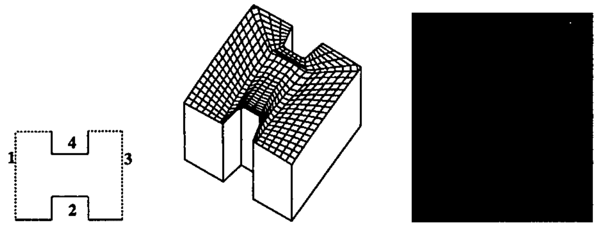

Fig. 14. Surface interpolation of two side concave area : pattern 3.

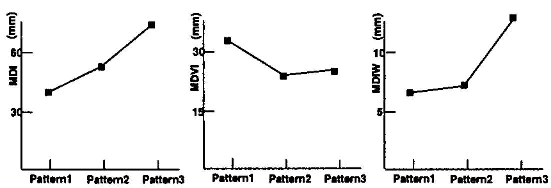

Fig. 15. Surface evaluation result.

From these results, two-sided concave cases require further study.

§4. Conclusion

We propose a non-four-sided interpolation method using a single NBG patch for a concave area. With this method, sub-patches are generated dividing at C^0 continuous point, component surfaces $\mathbf{S}^u$ and $\mathbf{S}^v$ are constructed by merging these sub-patches, which correct continuity.

We selected the 3 parameters to evaluate the smoothness of the surface streamlines. Selecting the four edges to minimize these parameters can generate a smooth surface. A one-sided concave area can be interpolated smoothly for complex cases. For two-sided concave case, evaluation parameter is effective, however, further research is necessary to improve the surface strain.

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J. Sone
Toshiba Corporation, 1 Toshiba-tyou, Futyuu, Tokyo, 183-0043, Japan
`junji.sone@toshiba.co.jp`

K. Konno
Ricoh Corporation, 1-1-17 Koishikawa, Bunkyou, Tokyo, 112-0002, Japan
`konno@src.ricoh.co.jp`

H. Chiyokura
Faculty if Environmental Information, Keio University,
5322 Endo, Fujisawa, Kanagawa, 252-0816, Japan
`chiyo@sfc.keio.ac.jp`